UNIT-5

5.1 VIEWING PIPELINE IN THREE DIMENSIONAL

Figure 12.2 shows the general processing steps for modeling and converting a world-coordinate description of a scene to device coordinates.

1. Once the scene has been modeled, world-coordinate positions are converted to viewing coordinates.
2. The viewing-coordinate system is used in graphics packages as a reference for specifying the observer viewing position and the position of the projection plane.
3. Next, projection operations are performed to convert the viewing-coordinate description of the scene to coordinate positions on the projection plane, which will then be mapped to the output device.
4. Objects outside the specified viewing limits are clipped from further consideration, and the remaining objects are processed through visible-surface identification.

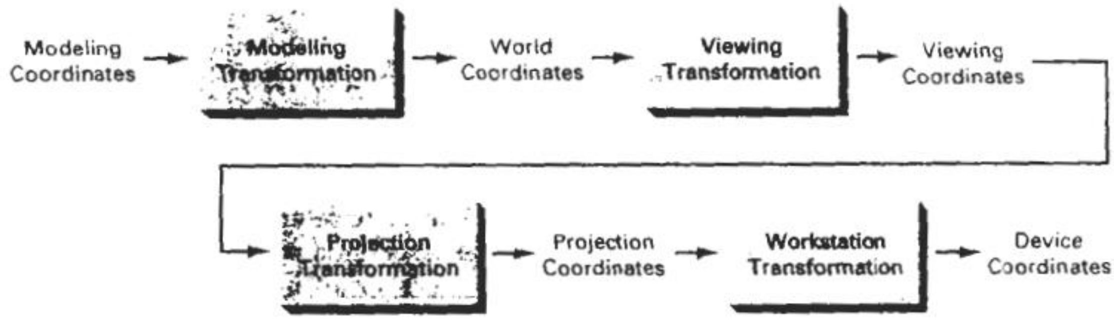

Figure 12-2
General three-dimensional transformation pipeline, from modeling coordinates to final device coordinates.

6.2 VIEWING COORDINATES

Generating a view of an object in three dimensions is similar to photographing the object. We can walk around and take its picture from any angle, at various distances, and with varying camera orientations.

The type and size of the camera lens determine which parts of the scene appear in the final picture.

Specifying the View Plane

We choose a particular view for a scene by first establishing the viewing-coordinate system, also called the view reference coordinate system, as shown in Fig. 12-3.

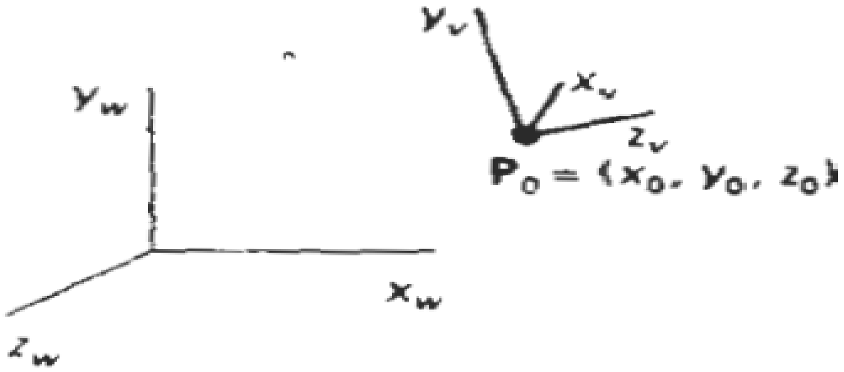

Figure 12-3
A right-handed viewing-coordinate system, with axes x_v, y_v, and z_v, relative to a world-coordinate scene.

A view plane, or projection plane, is then set up perpendicular to the viewing z_v axis

To establish the viewing-coordinate reference frame, we first pick a world coordinate position called the view reference point. This point is the origin of our viewing-coordinate system.

The transformation from World to Viewing Coordinates

This transformation sequence is

1. Translate the view reference point to the origin of the world-coordinate system.
2. Apply rotations to align the x_v, y_v, and z_v axes with the world x_w, y_w, and z_w axes, respectively.

If the view reference point is specified at world position (x_0, y_0, z_0), this point is translated to the world origin with the matrix transformation

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (12\text{-}1)$$

The rotation sequence can require up to three coordinate-axis rotations, depending on the direction we choose for **N**. In general, if **N** is not aligned with any world-coordinate axis, we can superimpose the viewing and world systems with the transformation sequence $\mathbf{R}_z \cdot \mathbf{R}_y \cdot \mathbf{R}_x$.

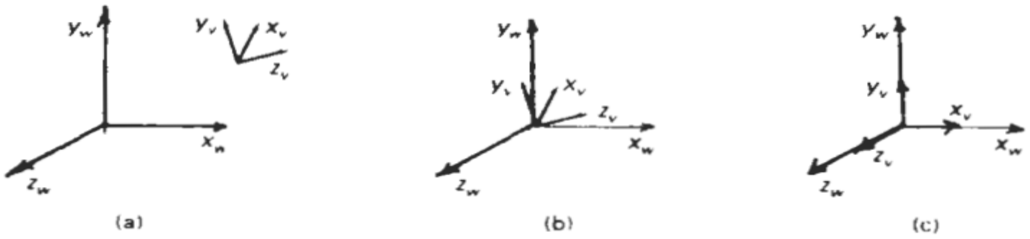

Figure 12-13
Aligning a viewing system with the world-coordinate axes using a sequence of translate-rotate transformations.

The complete world-to-viewing coordinate transformation matrix is obtained as the matrix product

$$\mathbf{M}_{WC,VC} = \mathbf{R} \cdot \mathbf{T} \tag{12-4}$$

This transformation is then applied to coordinate descriptions of objects in the scene to transfer them to the viewing reference frame.

6.3 PROJECTIONS

Once world-coordinate descriptions of the objects in a scene are converted to viewing coordinates, we can project the three-dimensional objects onto the two-dimensional view plane.

There are two basic projection methods.

1. In a parallel projection, coordinate positions are transformed to the vied plane along parallel lines, as shown in the , example of Fig. 12-14.
2. For a perspective projection (Fig. 12-15), object positions are transformed to the view plane along lines that converge to a point called the projection reference point (or center of projection)

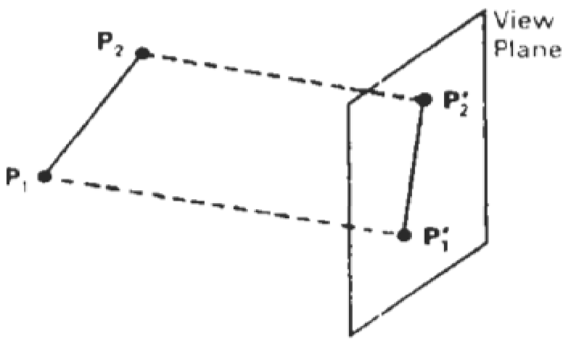

Figure 12-14
Parallel projection of an object to the view plane

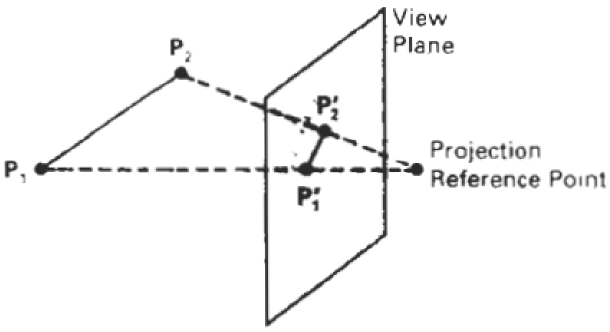

Figure 12-15
Perspective projection of an object to the view plane.

Accurate views of the various sides of an object are obtained with a parallel projection, but this does not give us a realistic representation of the appearance of a three-dimensional object.

A perspective projection, on the other hand, produces realistic views but does not preserve relative proportions.

A. PARALLEL PROJECTION

We can specify a **parallel projection with** a projection vector that defines the direction for the projection lines.

When **the projection is perpendicular to the view plane**, we have an orthographic parallel projection. Otherwise, we have an oblique parallel projection.

Figure 12-17
Orientation of the projection vector $\mathbf{V}_p$ to produce an orthographic projection (a) and an oblique projection (b).

Orthographic projections are most often used to produce the front, side, and top views of an object, as shown in Fig. 12-18 Front, side, and rear orthographic projections of an object are called **elevations,** and a top orthographic projection is called a **plane view.**

We can also form orthographic projections that display more than one face of an object. Such views are called **axonometric orthographic projections.**

The most commonly used **axonometric projection is** the isometric projection. We generate an isometric projection by aligning the projection plane so that it intersects each coordinate axis.

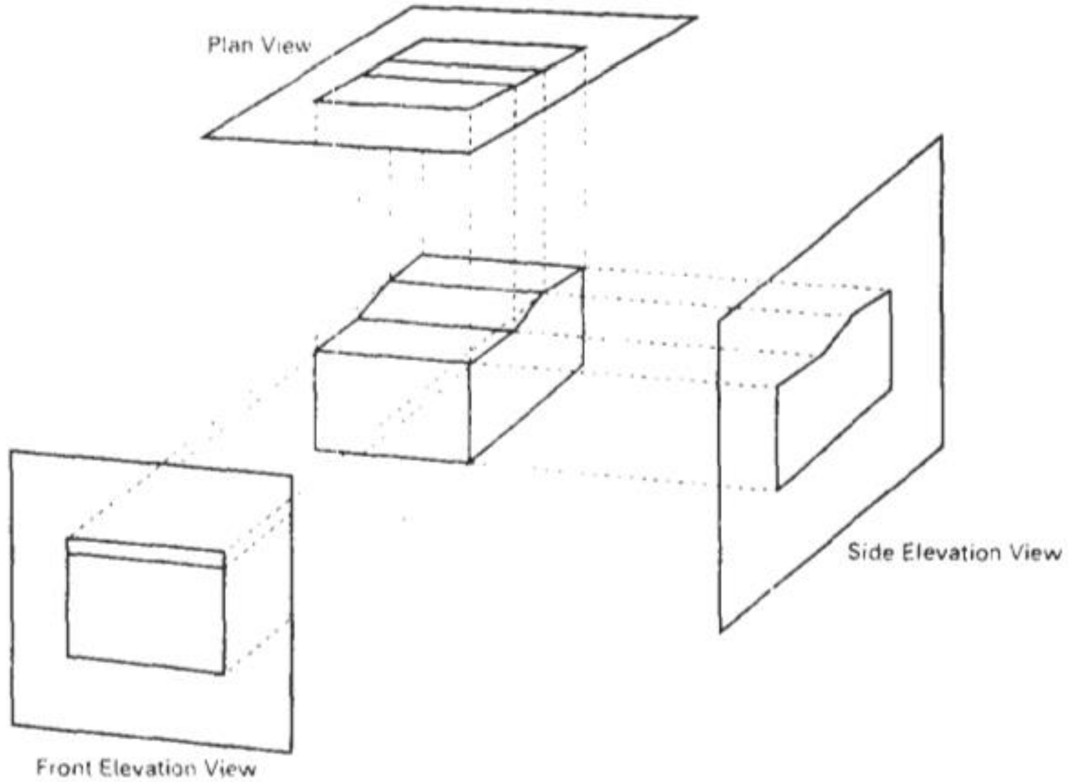

Figure 12-18
Orthographic projections of an object, displaying plan and elevation views.

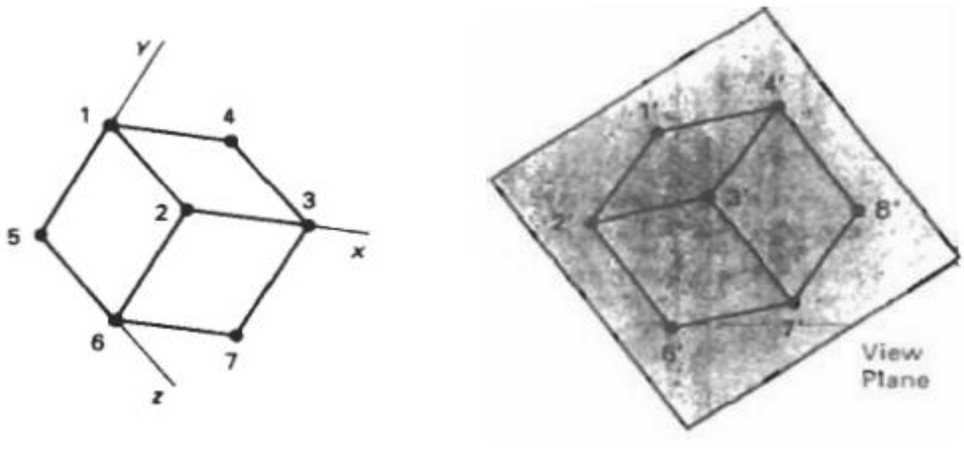

Figure 12-19
Isometric projection for a cube.

Transformation equations for an orthographic parallel projection are straight forward.

If the view plane is placed at position z, along the z, axis (Fig. 12-20), then any point (x, y, z) in viewing coordinates is transformed to projection coordinates as where the original z-coordinate value is preserved for the depth information needed in depth cueing and visible-surface determination procedures.

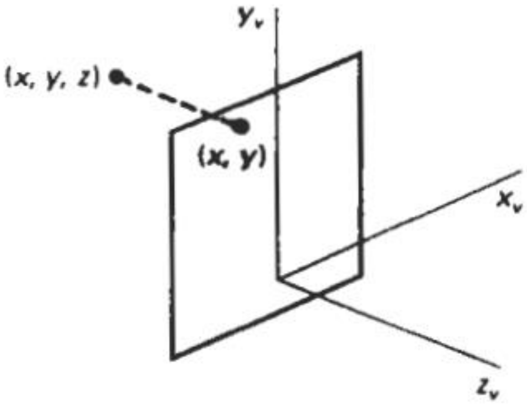

Figure 12-20
Orthographic projection of a point onto a viewing plane.

An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the projection plane. In some applications packages, an oblique projection vector is specified with two angles, α and ϕ, as shown in Fig. 12-21. Point (x, y, z) is projected to position (x_p, y_p) on the view plane. Orthographic projection coordinates on the plane are (x, y). The oblique projection line from (x, y, z) to (x_p, y_p) makes an angle α with the line on the projection plane that joins (x_p, y_p) and (x, y). This line, of length L, is at an angle ϕ with the horizontal direction in the projection plane. We can express the projection coordinates in terms of x, y, L, and ϕ as

$$x_p = x + L\cos\phi \qquad (12\text{-}6)$$
$$y_p = y + L\sin\phi$$

Length L depends on the angle α and the z coordinate of the point to be projected:

$$\tan\alpha = \frac{z}{L}$$

Thus,

$$L = \frac{z}{\tan\alpha}$$

where L_1 is the inverse of $\tan\alpha$, which is also the value of L when $z = 1$. We can then write the oblique projection equations 12-6 as

$$x_p = x + z(L_1 \cos\phi)$$
$$y_p = y + z(L_1 \sin\phi)$$

The transformation matrix for producing any parallel projection onto the $x_v y_v$ plane can be written as

$$\mathbf{M}_{\text{parallel}} = \begin{bmatrix} 1 & 0 & L_1 \cos\phi & 0 \\ 0 & 1 & L_1 \sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Common choices for angle ϕ are 30° and 45°, which display a combination view of the front, side, and top (or front, side, and bottom) of an object. Two commonly used values for α are those for which $\tan\alpha = 1$ and $\tan\alpha = 2$. For the first case, $\alpha = 45°$ and the views obtained are called **cavalier** projections.

When the projection angle α is chosen so that $\tan\alpha = 2$, the resulting view is called a **cabinet** projection. For this angle (≈63.4°), lines perpendicular to the viewing surface are projected at one-half their length. Cabinet projections appear more realistic than cavalier projections because of this reduction in the length of perpendiculars. Figure 12-24 shows examples of cabinet projections for a cube.

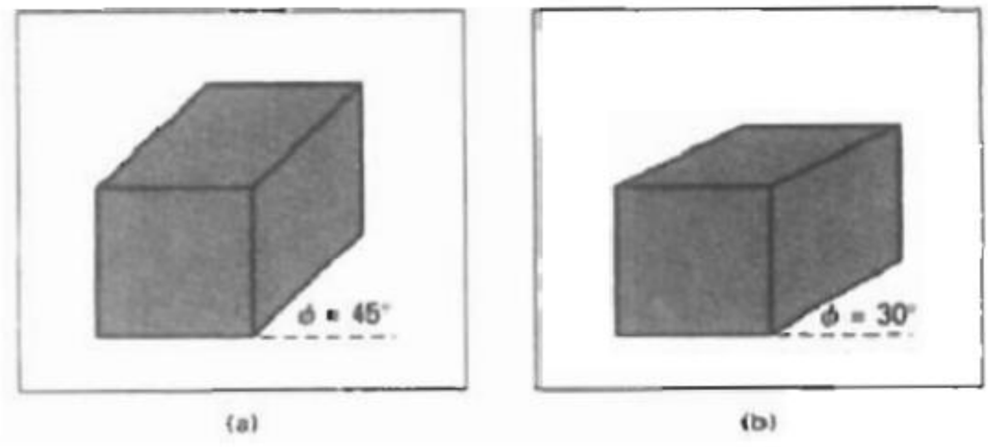

Figure 12-23
Cavalier projections of a cube onto a view plane for two values of angle ϕ.
Note: Depth of the cube is projected equal to the width and height.

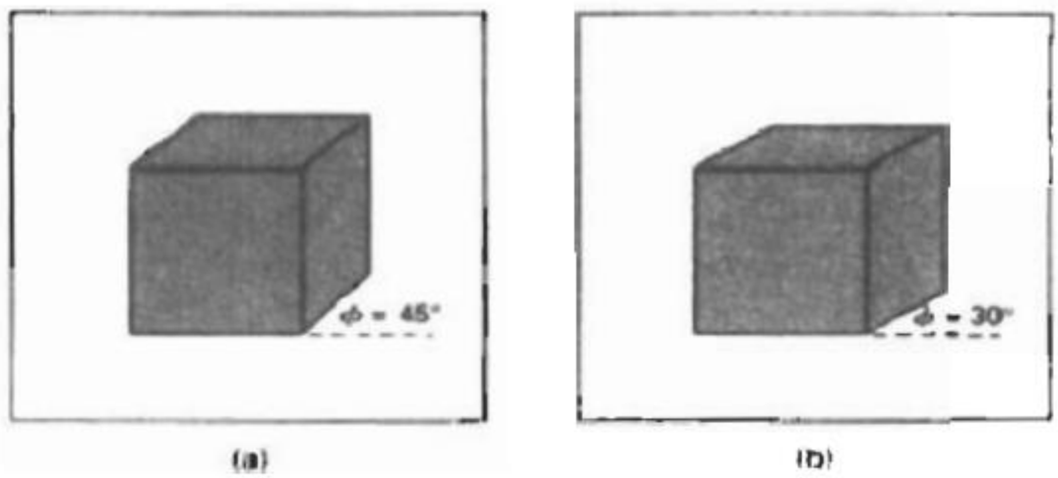

Figure 12-24
Cabinet projections of a cube onto a view plane for two values of angle ϕ. Depth is projected as one-half that of the width and height.

B. PERSPECTIVE PROJECTION

To obtain a perspective projection of a three-dimensional object, we transform points along projection lines that meet at the projection reference point.

Suppose we set the projection reference point at position z_{prp}, along the z_v axis, and we place the view plane at z_{vp} as shown in Fig. 12-25.

We can write equations describing coordinate positions along this perspective projection line in parametric form as

$$\begin{aligned} x' &= x - xu \\ y' &= y - yu \\ z' &= z - (z - z_{prp})u \end{aligned} \tag{12-11}$$

Parameter u takes values from 0 to 1, and coordinate position (x', y', z') represents any point along the projection line. When $u = 0$, we are at position $\mathbf{P} = (x, y, z)$. At the other end of the line, $u = 1$ and we have the projection reference point coordinates $(0, 0, z_{prp})$. On the view plane, $z' = z_{vp}$ and we can solve the z' equation for parameter u at this position along the projection line:

$$u = \frac{z_{vp} - z}{z_{prp} - z} \tag{12-12}$$

Substituting this value of u into the equations for x' and y', we obtain the perspective transformation equations

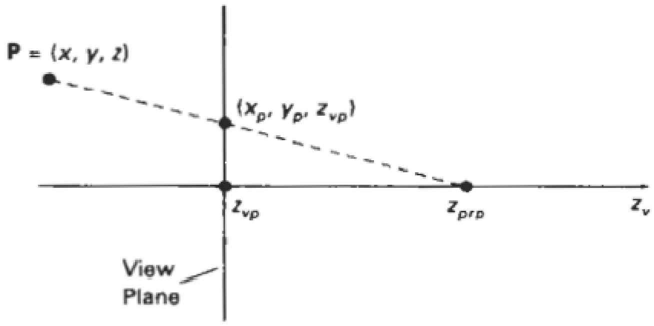

Figure 12-25
Perspective projection of a point **P** with coordinates (x, y, z) to position (x_p, y_p, z_{vp}) on the view plane.

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = x\left(\frac{d_p}{z_{prp} - z}\right)$$
$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = y\left(\frac{d_p}{z_{prp} - z}\right) \tag{12-13}$$

where $d_p = z_{prp} - z_{vp}$ is the distance of the view plane from the projection reference point.

Using a three-dimensional homogeneous-coordinate representation, we can write the perspective-projection transformation 12-13 in matrix form as

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -z_{vp}/d_p & z_{vp}(z_{prp}/d_p) \\ 0 & 0 & -1/d_p & z_{prp}/d_p \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{12-14}$$

In this representation, the homogeneous factor is

$$h = \frac{z_{prp} - z}{d_p} \tag{12-15}$$

and the projection coordinates on the view plane are calculated from the homogeneous coordinates as

$$x_p = x_h/h, \qquad y_p = y_h/h \tag{12-16}$$

In general, the projection reference point does not have to be along the z_v axis. We can select any coordinate position $(x_{prp}, y_{prp}, z_{prp})$ on either side of the view plane for the projection reference point, and we discuss this generalization in the next section.

There are a number of special cases for the perspective transformation equations 12-13. If the view plane is taken to be the uv plane, then $z_{vp} = 0$ and the projection coordinates are

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right) = x\left(\frac{1}{1 - z/z_{prp}}\right)$$
$$y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right) = y\left(\frac{1}{1 - z/z_{prp}}\right) \qquad (12\text{-}17)$$

And, in some graphics packages, the projection reference point is always taken to be at the viewing-coordinate origin. In this case, $z_{prp} = 0$ and the projection coordinates on the viewing plane are

$$x_p = x\left(\frac{z_{vp}}{z}\right) = x\left(\frac{1}{z/z_{vp}}\right)$$
$$y_p = y\left(\frac{z_{vp}}{z}\right) = y\left(\frac{1}{z/z_{vp}}\right) \qquad (12\text{-}18)$$

The vanishing point for any set of lines that are parallel to one of the principal axes of an object is referred to as a principal vanishing point.

We control the number of principal vanishing points (one, two, or three) with the orientation of the projection plane, and perspective projections are accordingly classified as one-point, two-point, or three-point projections.

The number of principal vanishing points in a projection is determined by the number of principal axes intersecting the view plane.

Figure 12-26 illustrates the appearance of one-point and two-point perspective projections for a cube. In. Fig. 12-26(b), the view plane is aligned parallel to the xy object plane so that only the object z axis is intersected.

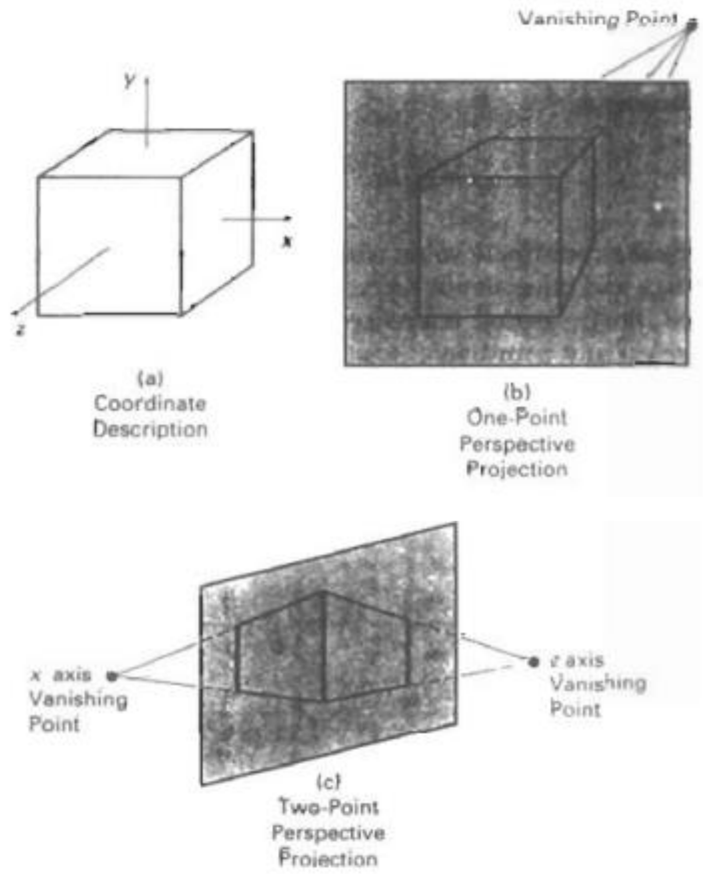

Figure 12-26
Perspective views and principal vanishing points of a cube for various orientations of the view plane relative to the principal axes of the object.